Mathematics benchmark assessment memorandum

1. $10 \times 2 + (6 - 4) \div 2 = ?$

   $20 + \frac{2}{2} = 21$

2. $\frac{3x}{4} + \frac{4x}{3} = ?$

   $\frac{3 \times 3x}{12} + \frac{4 \times 4x}{12} = \frac{9x + 16x}{12} = \frac{25x}{12}$

3. If $\ln\left(\frac{y}{1-y}\right) = a + bx$, solve for y.

   $\frac{y}{1-y} = e^{a+bx}$

   $y = e^{a+bx} - ye^{a+bx}$

   $y + ye^{a+bx} = e^{a+bx}$

   $y\left(1 + e^{a+bx}\right) = e^{a+bx}$

   $y = \frac{e^{a+bx}}{1 + e^{a+bx}}$

   Divide by e^{a+bx}

   $y = \frac{1}{1 + e^{-(a+bx)}}$

4. Simplify $\ln\left(\frac{A}{B}\right)$.

   $\ln\left(\frac{A}{B}\right) = \ln A - \ln B$

5. If $y = ae^{-bx}$, $\ln(y) = ?$

   $\ln y = \ln a - bx$

6. What is the maximum of the curve $y = ax - bx^2$?

$$\frac{d}{dx}\left\{ax - bx^2\right\} = 0$$
$$a - 2bx = 0$$
$$a = 2bx$$
$$x = \frac{a}{2b}$$

$$y = a\left(\frac{a}{2b}\right) - b\left(\frac{a}{2b}\right)^2$$
$$y = \frac{a^2}{2b} - \frac{a^2}{4b} = \frac{a^2}{b}\left(\frac{1}{2} - \frac{1}{4}\right)$$
$$y = \frac{a^2}{4b}$$

7. Solve $\frac{d}{dx}\left\{x^2 + 2y + 3xy\right\}$.

$$\frac{d}{dx}\left\{x^2 + 2y + 3xy\right\} = 2x + 3y$$

8. Solve $\frac{d}{dx}\left\{\frac{1}{\sqrt{x}}\right\}$.

Use the chain rule here.

Let $u = \sqrt{x} = x^{1/2}$

$$\frac{d}{dx}\left\{\frac{1}{\sqrt{x}}\right\} = \frac{dy}{du}\frac{du}{dx}$$
$$\frac{d}{du}\left\{u^{-1}\right\} \times \frac{d}{dx}\left\{x^{1/2}\right\}$$

$$-u^{-2} \times \frac{1}{2}x^{-1/2}$$
$$-\left(\sqrt{x}\right)^{-2} \times \frac{1}{2}x^{-1/2}$$
$$-\frac{1}{2x\sqrt{x}} = -\frac{1}{2x^{3/2}}$$

9. Solve $\frac{dy}{dx}\left\{xe^{-2x}\right\}$.

Use product rule and chain rule

$$= x\frac{dy}{dx}\left\{e^{-2x}\right\} + e^{-2x}\frac{dy}{dx}\left\{x\right\}$$
$$= xe^{-2x} \times -2 + e^{-2x} \times 1$$
$$= -2xe^{-2x} + e^{-2x}$$
$$= e^{-2x}\left(1-2x\right)$$

10. Solve $\frac{d}{dx}\left\{\ln x\right\}$.

$$\frac{d}{dx}\left\{\ln x\right\} = \frac{1}{x}$$

11. Solve $\frac{d}{dx}\left\{e^{-x}\right\}$.

$$\frac{d}{dx}\left\{e^{-x}\right\} = e^{-x}\frac{d}{dx}\left\{-x\right\} = -e^{-x}$$

12. Solve $\int mdx$

$$\int mdx = mx + C$$

13. What is the geometric mean of $\{y_1, y_2, ..y_n\}$?

$$\sqrt[n]{\prod_{i=1}^{n} y_i}$$

14. What is the standard error of the mean of $\{y_1, y_2, ..y_n\}$?

$$SE = s_{\bar{y}} = \frac{s^2}{n} = \frac{1}{n}\sqrt{\frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n}}$$

15. Explain what a null hypothesis is to your grandmother.

The null hypothesis is basically the idea/guess that there is no difference between measured phenomena and that any difference between them can be caused by chance alone.